Simultaneous Equations

Simu	ultaneous equations are two or more algebraic equations that														
share v	variables. There are two main types of simultaneous														
equation															
*.	linear simultaneous equations terms are raised to a power														
	that is not higher than one														
	$x^{2} + y^{2} = 25$														
	$ \begin{array}{c} x \\ y \\ y \\ z \\ x \\ y \\ z \\ y \\ z \\ y \\ z \\ y \\ z \\ z \\ z$														
· · · · *	auadratic cimultaneous equations contains terms that are														
	raised to a power that is not higher than two														
	a construction of the second a power that is riot inigher. That two is a														
	$x^2 + y^2 = 25$														
	x + y = x + 5														
How	vever, the type we will be focusing on is linear simultaneous														
equation	19														
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Iher	re are four ways to solve a simultaneous equation														
· · ·)	t elimination method														
	t substitution method														
	t matrix method														
· · · · · · · · · · · · · · · · · · ·	t graphically														
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Elimination Method

The word 'eliminate' means to remove. Therefore, when using this method, the goal is to remove variables until there is only one variable (unknown) left. This method involves adding the two equations together. Here are the steps to use to correctly using this method 1) label your equations 1 & 2 2) choose a variable to eliminate 3) make sure the coefficients of that variable are the same * if variables are not the same, find the LCM of each coefficient then multiply each equation accordingly 4) the signs of the coefficients must be opposite 5) operate by adding both equations 6) solve for the remaining variable 7) substitute the variable from #6 into one of the original equations in order to find the other variable 8) state your answer alphabetically in brackets & separated by a comma

Practice Questions													
x + y = -1 -x + y = 5	6x - 4y = 21 4x + 4y = 24	4x + 3y = 14 5x + 7y = 11											
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5x + 2y = 13 x + 2y = 9											
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Substitution Method

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	Worked Example	2
x + y = -1 $y = x + 5$	· · · · · · · · · · · · · · · · · · ·	. .
The second equ	ation is already solved for y . We in place of u in the first equation	will substitute
	x + y = -1	•
	$\mathbf{y} = \mathbf{x} + 5$	
· · · · · · · · ·	$\frac{1}{10} = \frac{1}{10} $	
	$\mathbf{y}_{1}^{*} - \mathbf{x}_{1} + \mathbf{y}_{2}^{*} - \mathbf{x}_{1} + \mathbf{y}_{2}^{*} \mathbf{y}_{1}^{*} \mathbf{y}_{2}^{*} \mathbf{y}_{1}^{*} \mathbf{y}_{2}^{*} \mathbf{y}_{2}^{*} \mathbf{y}_{1}^{*} \mathbf{y}_{2}^{*} \mathbf{y}_{1}^{*} \mathbf{y}_{2}^{*} \mathbf{y}_{1}^{*} \mathbf{y}_{2}^{*} \mathbf{y}_{2}^{*} \mathbf{y}_{1}^{*} \mathbf{y}_{2}^{*} y$	
Replace the y in	h the first equation with $X + 5$.	
	$\mathbf{v}_{\mathbf{i}} = \mathbf{x} + 5$	
	$\mathbf{x} + \mathbf{y} = -1$	
	x + (x + 3) = -1	
Solve the recul	$tin \sigma$ equation for \mathbf{V}	
	(X + (X + 5)) = -1	
	X + X + 3 = -1	
	2X + 3 = -1	
	2X = -1 - 5	
	2X = -6	
	x = -3	
• • • • • • •		

	Worked Exai	nple	• •	o o		•
			• •	• •		•
x + y = -1						
y = x + 5			• •	• •	• •	
			• •	• •	• •	
Solve the resulting	ng equation for x .			• •		•
	x + (x + 5) = -1		• •	• •		
	x + x + 5 = -1					
	$2\kappa + 5 = -1$			• •		
	$2_{\nu} - 1 - 5$		• •	•	• •	
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Substitute $x = -3$	into equation 2 to find t	ne value of y	· · ·	 	· ·	
Substitute $x = -3$	into equation 2 to find the second s	ne value of y	· · ·	· · ·	· · ·	•
Substitute $\kappa = -3$	into equation 2 to find the equation $y = x + 5$	ne value of y	· · ·	· · ·	· · ·	•
Substitute $x = -3$	into equation 2 to find the equation $y = x + 5$ y = -3 + 5	ne value of y	· · ·	· · · · · · · · · · · · · · ·	· · · · · · · · · · · · · · · · · · ·	
Substitute $x = -3$	into equation 2 to find the second s	ne value of y	· · · · · · · · · · · · · · · · · · ·	· ·	· · · · · · · · · · · · · · · · · · ·	
Substitute $\kappa = -3$	into equation 2 to find the equation $2 = x + 5$ y = -3 + 5 y = 2	ne value of y	· ·		· · · · · · · · · · · · · · · · · ·	•
Substitute $x = -3$	into equation 2 to find the equation 2 to find the equation 2 to find the equation $y = x + 5$ y = -3 + 5 y = 2	ne value of y	· · · · · ·			•
State your answe	into equation 2 to find the y = x + 5 y = -3 + 5 y = 2 er as an ordered pair.	ne value of y	· · · · · · · · · · · · · · · · · · ·			
Substitute $x = -3$ State your answe	into equation 2 to find the y = x + 5 y = -3 + 5 y = 2 er as an ordered pair. (-3, 2)					
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Simultaneous Equations - Substitution Method

Name :	Class :	Date :		
		Mark :	/8	%
1) Use a)	substitution to solve the following simultaneous equa $x + 3y = 5$ $x = 2$	tions	[8]	
b)	x = -4 x $-5y = -19$			
c)	y = 7x 4x + y = 55			
d)	$\begin{array}{rcl} x &=& y-5\\ x+& 3y &=& -1 \end{array}$			
e)	y = 2x + 7 3x + y = 17			
f)	y = x - 2 4x + 5y = 17			
g)	4x + 7y = 336 y = 6x + 2			
h)	7x + 3y = 32 6x + 4y = 36			

- 1. Solve 2x 3y = 115x + 2y = 18
- 2. Solve the simultaneous equations

$$2x + 3y = -3$$
$$3x - 2y = 28$$

$$6x - 2y = 33$$
$$4x + 3y = 9$$

4. Solve

$$x + 2y = 4$$
$$3x - 4y = 7$$

5. Solve the simultaneous equations

$$3x + 7y = 26$$
$$4x + 5y = 13$$

6. Solve the simultaneous equations.

$$5a + 3b = 9$$
$$2a - 3b = 12$$

BLIVEWORKSHEETS





Guidance

- 1. Read each question carefully before you begin answering it.
- 2. Don't spend too long on one question.
- 3. Attempt every question.
- 4. Check your answers seem right.
- 5. Always show your workings

Revision for this topic

www.corbettmaths.com/contents

Video 295 Video 296



$$5x + 3y = 41$$

 $2x + 3y = 20$

Do not use trial and improvement

2. Solve the simultaneous equations

5x + y = 113x - y = 9

Do not use trial and improvement

x = y =

$$x + 7y = 64$$

 $x + 3y = 28$

Do not use trial and improvement

4. Solve the simultaneous equations

4x - 4y = 24x - 4y = 3

Do not use trial and improvement

x = y =

$$2x + 4y = 26$$

 $3x - y = 4$

Do not use trial and improvement

8. Solve the simultaneous equations

$$3x + 2y = 16$$

 $2x - 3y = 2$

Do not use trial and improvement

x = y =

(4)

$$3x - 2y = 14$$

x + 2y = 10

Do not use trial and improvement

10. Solve the simultaneous equations

$$3x + 5y = 1$$

 $2x - 3y = 7$

Do not use trial and improvement

x = y =

(4)

$$3x - y = 23$$

 $2x + 3y = 8$

Do not use trial and improvement

12. Solve the simultaneous equations

$$2y - 5x = 9$$

 $4y + 3x = 5$

Do not use trial and improvement

x = y =

13. Find the coordinates where the straight lines below cross.

$$y - 3x = 3$$
$$x - 2y = 4$$

(.....) (4)

14. Solve the simultaneous equations

$$3a + c = 8$$

 $2a - c = 7$

Do not use trial and improvement

a = c =

are described in words. Here are a few helpful steps to use when															•																		
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Worked Examp	le i i i i i i i i i i i i i i i i i i i
Three oranges & 2 bananas together weigh 610a four bananas together weigh 1070g. What is the and 1 banana?	g. Five oranges & weight of 1 orange
Let x represent the mass of one orange, and y mass of one banana. X = orange $y = banana$ $identify$	the unknowns
Use the information to write your equations.	· · · · · · · · · · · · · ·
3x + 2y = 610	
5x + 4y = 1070	
Solve. 3x + 2y = 610 multiply 5x + 4y = 1070	$\frac{by}{by} - \frac{2}{b}$
-6x - 4y = -1220 5x + 4y = 1070	$\frac{Solve for y}{3x + 2y = 610}$
-x = -150	3(150) + 2y = 610 450 + 2y = 610
x = 150	2y = 610 - 450
	2y = 160
	y = 80
orange weighs 150g banana weighs 80g	
	· · · · · · · · · · · · · ·

- 3) Flying to Kampala with a tailwind a plane averaged 158 km/h. On the return trip the plane only averaged 112 km/h while flying back into the same wind. Find the speed of the wind and the speed of the plane in still air.
- 4) The school that Stefan goes to is selling tickets to a choral performance. On the first day of ticket sales the school sold 3 senior citizen tickets and 1 child ticket for a total of \$38. The school took in \$52 on the second day by selling 3 senior citizen tickets and 2 child tickets. Find the price of a senior citizen ticket and the price of a child ticket.
 - s = seniors c = child

#

- 5) The sum of the digits of a certain two-digit number is 7. Reversing its digits increases the number by 9. What is the number?
- 6) A boat traveled 210 miles downstream and back. The trip downstream took 10 hours. The trip back took 70 hours. What is the speed of the boat in still water? What is the speed of the current?

7) The state fair is a popular field trip destination. This year the senior class at High School A and the senior class at High School B both planned trips there. The senior class at High School A rented and filled 8 vans and 8 buses with 240 students. High School B rented and filled 4 vans and 1 bus with 54 students. Every van had the same number of students in it as did the buses. Find the number of students in each van and in each bus.

v = van b = bus

8) The senior classes at High School A and High School B planned separate trips to New York City. The senior class at High School A rented and filled 1 van and 6 buses with 372 students. High School B rented and filled 4 vans and 12 buses with 780 students. Each van and each bus carried the same number of students. How many students can a van carry? How many students can a bus carry?

v = van b = bus

9) Brenda's school is selling tickets to a spring musical. On the first day of ticket sales the school sold 3 senior citizen tickets and 9 child tickets for a total of \$75. The school took in \$67 on the second day by selling 8 senior citizen tickets and 5 child tickets. What is the price each of one senior citizen ticket and one child ticket?

s = seniors c = child

10) Matt and Ming are selling fruit for a school fundraiser. Customers can buy small boxes of oranges and large boxes of oranges. Matt sold 3 small boxes of oranges and 14 large boxes of oranges for a total of \$203. Ming sold 11 small boxes of oranges and 11 large boxes of oranges for a total of \$220. Find the cost each of one small box of oranges and one large box of oranges.

s = small boxes l = large boxes

11) A boat traveled 336 miles downstream and back. The trip downstream took 12 hours. The trip back took 14 hours. What is the speed of the boat in still water? What is the speed of the current?